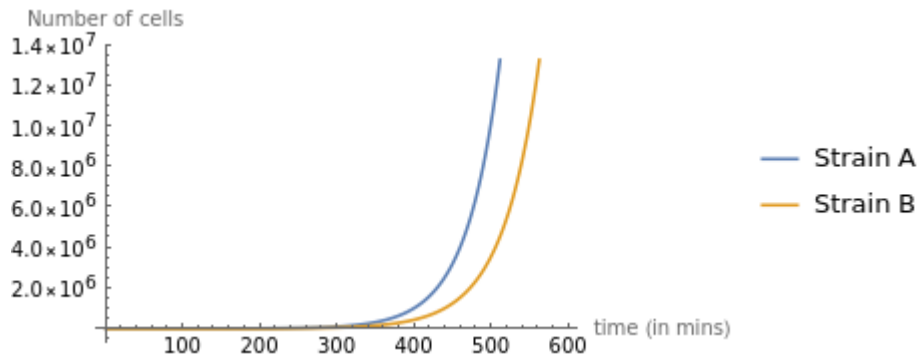


# Assignment 5

## 1) Exponential growth and competition

- a)  $n_a = n_a(t=0) * 2^{\frac{t}{t_{ad}}}$  (where,  $t_{ad}$  = doubling time of strain A)  
 $n_b = n_b(t=0) * 2^{\frac{t}{t_{bd}}}$  (where,  $t_{bd}$  = doubling time of strain B)



b)  $\frac{n_a}{n_b} = \frac{n_a(t=0)}{n_b(t=0)} * \frac{2^{\frac{t}{t_{ad}}}}{2^{\frac{t}{t_{bd}}}}$

Time (in hrs)	Ratio
5	1.87
10	3.52
20	12.43

c)  $\frac{n_a}{n_b} = 1000$

$$\frac{n_a}{n_b} = \frac{n_a(t=0)}{n_b(t=0)} * \frac{2^{\frac{t}{t_{ad}}}}{2^{\frac{t}{t_{bd}}}} = 2^{t * (\frac{1}{t_{ad}} - \frac{1}{t_{bd}})}$$

$$\Rightarrow t = \frac{\log_2 \frac{n_a}{n_b}}{(\frac{1}{t_{ad}} - \frac{1}{t_{bd}})}$$

$$\Rightarrow t = 54.81 \text{ hrs}$$

## 2) Optimality model 1

- a) On adding the translational inhibitor means  $\gamma$  decreases. Since  $c^* = \frac{1}{1 + \sqrt{\frac{\beta'}{\gamma}}}$ , with increase

in  $\gamma$ ,  $c^*$  decreases.

Institutively, if the people pressing the oranges at the juicer are less efficient (for some reason) then you need to allocate more resources to this side of the shop (R increases thus C decreases since  $C+R = 1$ ).

- b) On adding the inhibitor of C,  $\beta$  decreases. And since  $c^* = \frac{1}{1 + \sqrt{\frac{\beta'}{\gamma}}}$ , with decrease in  $\beta$ ,  $c^*$

increases.

Institutively, if the people peeling the oranges are less efficient (for some reason) then you need to allocate more resources to this side of the shop (C increases).

3)

$$\mu^* = \frac{\beta\gamma}{(\sqrt{\beta} + \sqrt{\gamma})^2}$$

$$c^* = \frac{\sqrt{\gamma}}{\sqrt{\beta} + \sqrt{\gamma}}$$

a) Growth rate of strain-1 =  $\frac{5*1}{(\sqrt{5} + \sqrt{1})^2} = 0.477 h^{-1}$

b) Optimal expression of C =  $\frac{\sqrt{1}}{\sqrt{5} + \sqrt{1}} = 0.309$

c) Now,  $\beta_{new} = 3 h^{-1}$ . Since strain-1 cannot adjust its expression for c, the growth rate of strain-1 in new condition will be =  $\frac{(1-c)\beta_{new}\gamma}{c(\beta_{new}-\gamma) + \gamma} = \frac{(1-0.309)*0.309*3*1}{0.309*(3-1) + 1} = 0.395 h^{-1}$ .

d) Growth rate of strain-2 =  $\frac{\beta_{new}\gamma}{(\sqrt{\beta_{new}} + \sqrt{\gamma})^2} = \frac{3*1}{(\sqrt{3} + 1)^2} = 0.401 h^{-1}$ .

$$\% \text{ decrease in growth of strain - 1} = \frac{0.401 - 0.395}{0.401} * 100 = 1.5\%$$

e) Let  $n(t)$  = number of cells and  $\mu$  = growth rate of cells then,

$$n_1(t) = n_0 * e^{\mu_1 t}$$

$$n_2(t) = n_0 * e^{\mu_2 t}$$

$$\Rightarrow t = \frac{1}{\mu_1 - \mu_2} * \ln \frac{n_1(t)}{n_2(t)}$$

$$\Rightarrow t = \frac{1}{0.401 - 0.395} * \ln \frac{1000}{1} = 1151.2 \text{ hrs}$$

It takes around 1151.2 hrs for strain-2 to be 1000 times more abundant than strain-1.