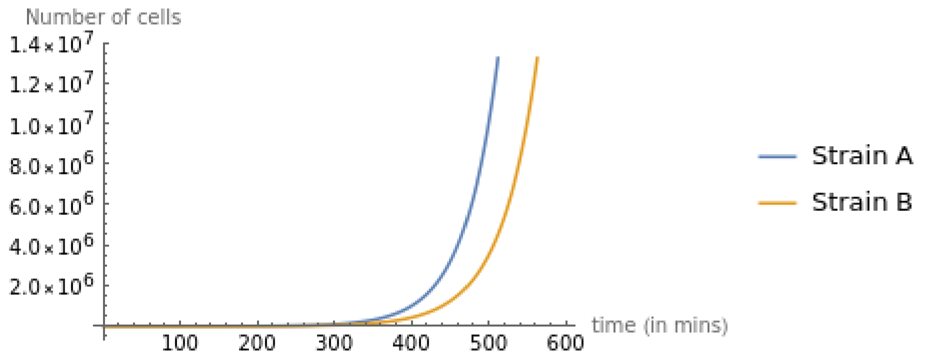
Assignment 5

1) Exponential growth and competition

a) $n_a = n_a(t=0) * 2^{\frac{t}{t_{ad}}}$ $(where,\ t_{ad} = doubling\ time\ of\ strain\ A)$

$n_b = n_b(t=0) * 2^{\frac{t}{t_{bd}}}$ $(where,\ t_{bd} = doubling\ time\ of\ strain\ B)$



b) $\frac{n_a}{n_b} = \frac{n_a(t=0)}{n_b(t=0)} * \frac{2^{\frac{t}{t_{ad}}}}{2^{\frac{t}{t_{bd}}}}$

Time (in hrs)	Ratio
5	1.87
10	3.52
20	12.43

c) $\frac{n_a}{n_b} = 1000$

$$\frac{n_a}{n_b} = \frac{n_a(t=0)}{n_b(t=0)} * \frac{2^{\frac{t}{t_{ad}}}}{2^{\frac{t}{t_{bd}}}} = 2^{t*(\frac{1}{t_{ad}} - \frac{1}{t_{bd}})}$$

$$\Rightarrow t = \frac{\log_2 \frac{n_a}{n_b}}{(\frac{1}{t_{ad}} - \frac{1}{t_{bd}})}$$

$$\Rightarrow t = 54.81\ hrs$$

2) Optimality model 1

a) On adding the translational inhibitor means γ decreases. Since $c^* = \frac{1}{1+\sqrt{\frac{\beta}{\gamma}}}$, with decrease in γ, c^* decreases.

Institutively, if the people pressing the oranges at the juicer are less efficient (for some reason) then you need to allocate more resources to this side of the shop (R increases thus C decreases since C+R = 1).

b) On adding the inhibitor of C, β decreases. And since $c^* = \frac{1}{1+\sqrt{\frac{\beta}{\gamma}}}$, with decrease in β, c^* increases.

Institutively, if the people peeling the oranges are less efficient (for some reason) then you need to allocate more resources to this side of the shop (C increases).

3)

$$\mu^* = \frac{\beta\gamma}{(\sqrt{\beta} + \sqrt{\gamma})^2}$$

$$c^* = \frac{\sqrt{\gamma}}{\sqrt{\beta} + \sqrt{\gamma}}$$

a) Growth rate of strain-1 = $\frac{5*1}{(\sqrt{5}+\sqrt{1})^2} = 0.477\ h^{-1}$

b) Optimal expression of C = $\frac{\sqrt{1}}{\sqrt{5}+\sqrt{1}} = 0.309\ h^{-1}$

c) Now, $\beta_{new} = 3\ h^{-1}$. Since strain-1 cannot adjust its expression for c, the growth rate of stain-1 in new condition will be = $\frac{(1-c)c\ \beta_{new}\gamma}{c(\beta_{new}-\gamma)+\gamma}$ = $\frac{(1-0.309)*0.309*3*1}{0.309*(3-1)+1} = 0.395\ h^{-1}$.

d) Growth rate of strain-2 = $\frac{\beta_{new}\gamma}{(\sqrt{\beta_{new}}+\sqrt{\gamma})^2} = \frac{3*1}{(\sqrt{3}+1)^2} = 0.401\ h^{-1}$.

$$\%\ decrease\ in\ growth\ of\ strain - 1 = \frac{0.401 - 0.395}{0.401} * 100 = 1.5\%$$

e) Let $n(t)$ = number of cells and μ = growth rate of cells then,

$$n_1(t) = n_0 * e^{\mu_1 t}$$
$$n_2(t) = n_0 * e^{\mu_2 t}$$
$$\Rightarrow t = \frac{1}{\mu_1 - \mu_2} * \ln\frac{n_1(t)}{n_2(t)}$$
$$\Rightarrow t = \frac{1}{0.401 - 0.395} * \ln\frac{1000}{1} = 1151.2\ hrs$$

It takes around 1151.2 hrs for strain-2 to be 1000 times more abundant than strain-1.