Assignment #5

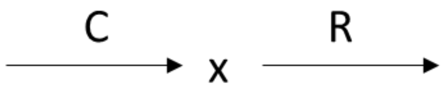
1. Exponential growth and competition

Two bacterial strains grow together in the same shaking flask, such that they are well mixed. They both start at the same cell number n_A=n_B=100. Strain A has a doubling time of 30 minutes. Strain B has a doubling time of 33 minutes. Assume that growth is unlimited (no saturation)

1. Plot the number of cells for each strain from time t=0h until time t = 10h.
2. What is the ration of cells n_A/n_B after 5 hours? 10 hours? 20 hours?
3. How long does it take until there are 1000 times more cells of strain A than of strain B?

2. Optimality model 1

Use the C/R resource allocation model discussed in the lecture to answer the following questions:



In the model, C is the fraction of the proteome allocated to nutrient uptake. R is the fraction of the proteome allocated to ribosomes. The sum of C+R = 1. x is a biosynthetic intermediate produced by C and consumed by R to produce biomass (think of it as amino acids). β is the catalytic rate of C, γ is the catalytic rate of R.

Assuming the following production and consumption of x, and steady-state for x ($x' = 0$),

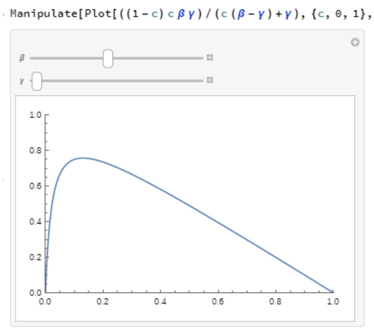
$x'[t] == \beta \frac{1}{1+x} C - \gamma \frac{x}{1+x} R$

one can calculate the growth rate as follows:

$$\mu = \gamma \frac{x}{1+x} R = \frac{(1-c)c\beta\gamma}{c(\beta-\gamma)+\gamma}$$

The following code plots the model in Mathematica with sliders to change the parameters beta and gamma.

Manipulate[Plot[((1-c)c β γ)/(c(β-γ)+γ),{c,0,1}, PlotRange->{{0,1},{0,1}}],{{β,10},0,100},{{γ,1},0,100}]



2.1 How does the best allocation to C (that maximizes the growth rate) change when you add a translational inhibitor to the growth medium that partially inhibits ribosomal activity?
2.2 What happens if you add an inhibitor of C?

3. Optimality model 2: The selective benefit of controlling C
Use the mode from part 2 for the following section.

The expression of c that maximizes the growth rate can be determined by calculating the peak of the above function as follows:

$$\text{Solve}[D[\frac{(1-c)c\beta\gamma}{c(\beta-\gamma)+\gamma}, c] == 0, c]$$

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Solve[D[((1-c)c β γ)/(c(β-γ)+γ),c]==0,c]
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Solving this equation shows that the optimal expression c* of C is c* = $\frac{\sqrt{\gamma}}{\sqrt{\beta}+\sqrt{\gamma}}$ note that (there is also a negative solution for the equation, but negative concentrations are not meaningful in a biological context)

The corresponding maximal growth rate μ* at c* is

$$\frac{(1-c)c\,\beta\,\gamma}{c(\beta-\gamma)+\gamma} /.\, c \to \frac{\sqrt{\gamma}}{\sqrt{\beta}+\sqrt{\gamma}} //\text{Simplify}$$

$$\mu *= \frac{\beta\,\gamma}{(\sqrt{\beta}+\sqrt{\gamma})^2}$$

The bacterial strain 1 has evolved under conditions where $\beta = 5/h$ and $\gamma = 1$/h all the time. It has therefore lost all of its regulatory network controlling c and therefore cannot regulate C at all anymore.

3.1 What is the growth rate of strain 1?
3.2 What is the optimal expression of C that strain 1 expresses?

The environment of the strain changes, such that now $\beta = 3$ and $\gamma = 1$.

3.3 What is the growth rate of strain 1 under this new condition assuming the model above and that strain 1 does not change the expression of c in response to the change in β?
3.4 How much slower does the strain 1 grow compared to strain 2 that has retained the regulatory mechanism and always reach the maximal growth rate under each condition for all parameters β and γ?
3.5 How long does it take for strain 2 to be 1000 times more abundant than strain 1 if they both start with the same cell number?